to obtain the final result. We adopted this method because we could use a conventional refinement program. If a program handling two or more sets of intensity data had been available, we could have simultaneously refined the $\mathrm{Co} / \mathrm{Zn}$ and $\mathrm{Ni} / \mathrm{Zn}$ ratios.
Even where the atomic numbers of the cations are considerably different, the two-wavelength anoma-lous-dispersion method would be useful for determining ternary-system cation distributions. One set of intensity data measured at a certain wavelength may determine a ternary-system cation distribution to some extent, but the errors would be large. To reduce the errors, two sets of intensity data measured at two different wavelengths using the anomalous-dispersion effect need to be collected.
Our two-wavelength method may be extended to other combinations such as neutron and X-ray diffraction. This combination is effective for determining a ternary-system cation distribution because the atomic scattering factors for neutron diffraction are different from those for X-ray diffraction. Mössbauer spectroscopy in combination with X-ray diffraction could also be used to determine a ternary-system cation distribution if the system includes iron. These two independent methods are effective for a ternary system.

The atoms ( $\mathrm{Co}, \mathrm{Ni}$ and Zn ) in the $M 1$ or $M 2$ sites should have different positions from each other, because these sites are general positions. If we have four sets of intensity data (e.g. $\mathrm{Co}, \mathrm{Ni}$ and Zn absorption edges, and Mo $K \alpha$ data), we can refine the positional parameters of each atom. Sasaki \& Tsukimura (1987) reported such a procedure to determine the positional parameters of each atom in $(\mathrm{Co}, \mathrm{Ni}, \mathrm{Zn}) \mathrm{SiO}_{3}$ pyroxene.

## References

Ghose, S., Wan, C. \& Okamura, F. P. (1975). Acta Cryst. A31, S76.
International Tables for X-ray Crystallography (1974). Vol. IV, Table 2.2B. Birmingham: Kynoch Press. (Present distributor Kluwer Academic Publishers, Dordrecht.)
Morimoto, N., Nakahma, Y., Syono, Y., Akimoto, S. \& Matsui, Y. (1975). Acta Cryst. B31, 1041-1049.
SASAKI, S. (1984). KEK Report 83-22. National Laboratory for High Energy Physics, Japan.
SASAKI, S. (1987). KEK Internal 87-3. National Laboratory for High Energy Physics, Japan.
Sasaki, S. \& Tsukimura, K. (1987). J. Phys. Soc. Jpn, 56, 437-440.
Tokonami, M. (1965). Acta Cryst. 19, 486.
Tsukimura, K. (1989). Mineral. J. 14, 323-337.
Yakel, H. L. (1980). J. Phys. Chem. Solids, 41, 1097-1104.
Yakel, H. L. (1983). Acta Cryst. B39, 20-28.

# Structure of $\mathrm{Pb}_{5} \mathrm{Cr}_{3} \mathbf{F}_{\mathbf{1 9}}$ at $\mathbf{2 9 5} \mathbf{K}$, Polarization Reversal and the 555 K Phase Transition 

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#### Abstract

Lead chromium fluoride, $\mathrm{Pb}_{5} \mathrm{Cr}_{3} \mathrm{~F}_{19}$, with $M_{r}=$ 1552.9 is a new material with ferroelectric attributes that crystallizes in the tetragonal system. At $T=$ 295 K , the space group is $I 4 \mathrm{~cm}$ with $a=14.384$ (5) and $c=7.408$ (2) $\AA, V=1532.8$ (7) $\AA^{3}, Z=4, D_{m}=$ $6.67(5), \quad D_{x}=6.729 \mathrm{~g} \mathrm{~cm}^{-3}$. For $\lambda(\mathrm{Mo} \mathrm{K} \alpha)=$

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$0.71073 \AA, \quad \mu=57.33 \mathrm{~mm}^{-1} . \quad F(000)=2612$. The material undergoes a first-order phase transition at 555 K . The structure was determined from 7599 ( 1410 independent) $F_{m}^{2} \geq 4 \sigma\left(F_{m}^{2}\right)$ with $(\sin \theta) / \lambda \leq$ $1.029 \AA^{-1}$; least-squares refinement on $w F_{m}{ }^{2}$ led to $R\left(F_{m}\right)=0.044$ with $R_{\text {int }}\left(F_{m}\right)=0.040$. No atom is more than $0.72 \AA$ from the hypothetical paraelectric state, hence the crystal is structurally ferroelectric.

[^0]The mean atomic displacement by Cr from the zero spontaneous-polarization location corresponds to a predicted phase-transition temperature of $635(30) \mathrm{K}$, in satisfactory agreement with the experimental temperature $T_{c}=555 \mathrm{~K}$. The Cr atoms form two symmetry-independent distorted octahedra, with average $\mathrm{Cr}-\mathrm{F}$ distances of 1.92 (3) and 1.85 (1) $\AA$ respectively. One Pb atom is nine-coordinated, occupying a distorted tricapped trigonal prism with average $\mathrm{Pb}-\mathrm{F}=2.69$ (20) $\AA$. The other Pb atom is ten-coordinated with a coordination polyhedron based on a bisdisphenoid; six F atoms are at an average distance of $2 \cdot 42$ (8) $\AA$, with four additional $F$ atoms at 3.024 (1) $\AA$.

## Introduction

Ferroelectric attributes have recently been recognized in three related families, for which the archetypes are $\mathrm{SrAlF}_{5}$ (von der Mühll, Andersson \& Galy, 1971; Ravez, von der Mühll \& Hagenmuller, 1975; Abrahams, Ravez, Simon \& Chaminade, 1981), $\mathrm{Sr}_{3}\left(\mathrm{FeF}_{6}\right)_{2}$ (von der Mühll, 1974; Abrahams, Ravez, Canouet, Grannec \& Loiacono, 1984) and $\mathrm{Pb}_{5} \mathrm{~W}_{3} \mathrm{O}_{9} \mathrm{~F}_{10}$ (Ravez, Abrahams, Marsh, Arquis \& Chaminade, 1985; Arquis-Canouet, Ravez, Chaminade, Hagenmuller, Abrahams \& Marsh, 1986; Abrahams, Marsh \& Ravez, 1987). These attributes include a higher than room-temperature phase transition, a nonlinear dielectric susceptibility, a dielectric permittivity maximum and a dielectric loss minimum at the phase transition in addition to an atomic arrangement capable of undergoing displacements that give rise to a reversible spontaneous polarization. All three families are tetragonal, with the following relationships among their unit cells:

$$
\begin{gathered}
a\left(\mathrm{SrAlF}_{5}\right) \simeq a\left[\mathrm{Sr}_{3}\left(\mathrm{FeF}_{6}\right)_{2}\right] / 2^{1 / 2} \simeq a\left(\mathrm{~Pb}_{5} \mathrm{~W}_{3} \mathrm{O}_{9} \mathrm{~F}_{10}\right) \\
c\left(\mathrm{SrAlF}_{5}\right) \simeq c\left[\mathrm{Sr}_{3}\left(\mathrm{FeF}_{6}\right)_{2}\right] \simeq 2 c\left(\mathrm{~Pb}_{5} \mathrm{~W}_{3} \mathrm{O}_{9} \mathrm{~F}_{10}\right) .
\end{gathered}
$$

The nonlinear optical material $\mathrm{Pb}_{5} \mathrm{Cr}_{3} \mathrm{~F}_{19}$, initially prepared as a possible isomorph of $\mathrm{Pb}_{5} \mathrm{~W}_{3} \mathrm{O}_{9} \mathrm{~F}_{10}$, has been reported by Arquis-Canouet, Ravez \& Abrahams (1986) to undergo a first-order phase transition at 555 K . The $a$ axis expands and the $c$ axis contracts abruptly at the Curie temperature, both by about $0 \cdot 6 \%$. The uniaxial indicatrix sharply reverses sign from negative to positive on heating through the phase transition, with pronounced hysteresis on cooling. Phase transitions have also been reported in ten other members of the $\mathrm{Pb}_{5} \mathrm{Cr}_{3} \mathrm{~F}_{19}$ family (Ravez, Arquis, Grannec, Simon \& Abrahams, 1987). Despite their similarity in lattice constants, $\mathrm{Pb}_{5} \mathrm{Cr}_{3} \mathrm{~F}_{19}$ and $\mathrm{Pb}_{5} \mathrm{~W}_{3} \mathrm{O}_{9} \mathrm{~F}_{10}$ crystallize in different space groups, with atomic arrangements that also are different. The structure of $\mathrm{Pb}_{5} \mathrm{Cr}_{3} \mathrm{~F}_{19}$ has now been determined at room temperature and shown to meet the structural
criteria for ferroelectricity (Abrahams, 1988). The results are presented below and compared with those reported for $\mathrm{Pb}_{5} \mathrm{~W}_{3} \mathrm{O}_{9} \mathrm{~F}_{10}, \mathrm{Sr}_{3}\left(\mathrm{FeF}_{6}\right)_{2}$ and $\mathrm{SrAlF}_{5}$.

## Experimental

The preparation and crystal growth of $\mathrm{Pb}_{5} \mathrm{Cr}_{3} \mathrm{~F}_{19}$ have been reported previously by Arquis-Canouet, Ravez \& Abrahams (1986) together with the density and lattice constants, the latter over the temperature interval $295-610 \mathrm{~K}$. A transparent, regular prismatic crystal with a brilliant, deep-green color and dimensions $0.084 \times 0.112 \times 0.252 \mathrm{~mm}$ (linear e.s.d. about $1 \%$ ) was mounted on a Pyrex capillary. All integrated intensities, within the volume of reciprocal space bounded by $h-13 \rightarrow 29, k-29 \rightarrow 29, l-15 \rightarrow$ 15 and $(\sin \theta) / \lambda \leq 1.029 \AA^{-1}$, were measured using an Enraf-Nonius CAD-4 diffractometer. Mo $K \alpha$ radiation, diffracted by a graphite monochromator, was used with $\omega-2 \theta$ scans over an angular range $0.85^{\circ}+0.35^{\circ} \tan \theta$. The maximum time spent on a scan was 240 s for a counting-statistics standard deviation objective of $\sigma_{c}(I) / I \leq 0.01$. Six standard reflections were measured at 6 h intervals during a total exposure time of 1176 h . A linear intensity decline with time $t(\mathrm{~h})$ was observed and corrected using the fitted equation $I_{t}=I_{o}\left(1-6.24 \times 10^{-5} t\right)$.

A total of 16828 integrated intensities, including standards, were measured at 295 K and further corrected for Lorentz, polarization and absorption effects, the latter by numerical integration. Transmission factors varied from 0.0057 to 0.0955 . The 7599 intensities that were measured more than once and that exceeded $4 \sigma_{c}(I)$ were averaged in point group 4 mm , with separated Friedel pairs, to give 3575 structure amplitudes $\left\langle F_{m}^{2}\right\rangle$ of which 1410 had $\left\langle F_{m}^{2}\right\rangle \geq$ $4 \sigma\left\langle F_{m}^{2}\right\rangle$. The variances $\sigma^{2}\left\langle F_{m}^{2}\right\rangle$ were estimated, by a variation of Abrahams, Bernstein \& Keve's (1971) method, as the larger of $V_{1}$ or $V_{2}$ where $V_{1}$ is the sum of the variances derived from counting statistics ( $\sigma_{c}$ ), the absorption correction, and the variations among the standard reflections; $V_{2}$ is the estimated variance of the weighted-mean amplitude $\left\langle F_{m}^{2}\right\rangle$ of a form. The internal agreement factors are $R_{\text {int }}\left(F_{m}\right)=0.040$ and $w R_{\text {int }}\left(F_{m}\right)=0.037$. All agreement factors are defined as in Notes for Authors (1983).

The diffraction symmetry of $\mathrm{Pb}_{5} \mathrm{Cr}_{3} \mathrm{~F}_{19}$ is $4 / \mathrm{mmm}$, with systematic absences $h k l$ for $h+k+l$ odd and 0 kl for $l,(k)$ odd. Possible point groups are limited to 4 mm (space group $I 4 \mathrm{~cm}$ ) and $\overline{4} \mathrm{~m} 2$ (space group $\overline{4} \mathrm{c} c 2$ ) by the observation of second-harmonic generation (Abrahams et al., 1984) in the material which at that time was taken as having the composition $\mathrm{Pb}_{3}\left(\mathrm{CrF}_{6}\right)_{2}$. Point group $\overline{4} m 2$ is nonpolar and may thus be eliminated if the material is ferroelectric. The structure solution described below establishes 14 cm as the most likely space group.

Table 1. $\mathrm{Pb}_{5} \mathrm{Cr}_{3} \mathrm{~F}_{19}$ atomic coordinates and equivalent isotropic temperature factors $\left(\AA^{2}\right)$ with e.s.d.'s

Coordinates without parentheses were not varied. The $z$ coordinate for $\mathrm{Pb}(1)$ defines the origin. For atoms $\mathrm{Cr}(1)$ and $\mathrm{F}(4)-\mathrm{F}(6)$ the position $\frac{1}{2}-x, x, z$ is given. $U_{\mathrm{eq}}=\frac{1}{3}\left(U_{11}+U_{22}+U_{33}\right)$.

|  | $x$ | $y$ | $z$ | $U_{\text {cq }}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Pb}(1)$ | 0.4264 (2) | $0 \cdot 2727$ (2) | 0 | 0.017 (1) |
| $\mathrm{Pb}(2)$ | 0 | 0.5 | 0.1508 (8) | 0.045 (2) |
| $\mathrm{Cr}(1)$ | $0 \cdot 1632$ | $0 \cdot 3368$ (4) | -0.0107 (12) | 0.011 (2) |
| $\mathrm{Cr}(2)$ | 0 | 0 | 0.1799 (6) | 0.011 (4) |
| $\mathrm{F}(1)$ | 0.1085 (11) | 0.2488 (11) | -0.1726 (31) | 0.045 (9) |
| F(2) | 0.1984 (8) | 0.4309 (8) | $0 \cdot 1655$ (16) | 0.022 (4) |
| F(3) | -0.0569 (10) | $0 \cdot 1153$ (10) | 0.6875 (37) | 0.070 (16) |
| F(4) | 0.2523 | 0.2477 (8) | 0.0571 (27) | 0.022 (6) |
| F(5) | 0.3856 | $0 \cdot 1144$ (6) | 0.0195 (37) | 0.021 (5) |
| F(6) | 0.0720 | 0.4280 (10) | -0.0828 (44) | 0.066 (16) |
| F (7) | 0 | 0 | 0.4323 (71) | 0.51 (1) |

## Solution and refinement of the structure

The location of two symmetry-independent Pb and two symmetry-independent Cr atoms was deduced from a three-dimensional Patterson map. Subsequent difference electron density maps gave the location of seven symmetry-independent F atoms, see Table 1. The multiplicity and Wyckoff letter for each atom is given in Table 2. Not all coordinates correspond to the first set given in International Tables for Crystallography (1987) but are convenient in representing the coordination polyhedra.

The structure model was refined by minimizing the function $\sum w\left(\left\langle F_{m}^{2}\right\rangle-F_{o}^{2}\right)^{2}$ with weights $w=1 / \sigma^{2}\left\langle F_{m}^{2}\right\rangle$ (see above for the estimation method). The $1410\left\langle F_{m}^{2}\right\rangle$ $\geq 4 \sigma\left\langle F_{m}^{2}\right\rangle$ amplitudes were used in the refinements but all values of the indicators given below are based on $\left\langle F_{m}\right\rangle$. A model in which the Pb and Cr atoms were allowed to vibrate harmonically with anisotropic displacement parameters while F atoms were treated isotropically converged to $R=0.072, w R=0.066$ and $S=1 \cdot 655$. All independent atoms refined rapidly to their final positions with the exception of $\mathrm{F}(7)$ : this atom consistently appeared to have an unusually high displacement factor, possibly indicating an unresolved disorder about the $c$ axis.

The possibility of anharmonic thermal vibrations by the Pb and Cr atoms in $\mathrm{Pb}_{5} \mathrm{Cr}_{3} \mathrm{~F}_{19}$ was investigated in a model that finally included all tensors to fifth order for $\mathrm{Pb}(1)$ and $\mathrm{Cr}(2)$ and fourth order for $\mathrm{Pb}(2)$ and $\mathrm{Cr}(1)$, with probability density functions given by a Gram-Charlier expansion in a locally modified verison of the $O R F L S$ least-squares program of Busing, Martin \& Levy (1973). All F atoms except $F(7)$ were modelled with anisotropic displacement parameters. Anharmonic tensors to fourth order were included for $F(7)$ in addition to the anisotropic displacement parameters. The final model converged to $R=0.044, w R=0.042$ and $S=$ $1 \cdot 140$. All atomic positional and displacement parameters had ratios $\Delta / \sigma<0.19$. The resulting

Table 2. Polar coordinates and atomic displacements necessary for $\mathrm{Pb}_{5} \mathrm{Cr}_{3} \mathrm{~F}_{19}$ to become paraelectric
$\Delta z=\left(z^{\prime}-z^{\prime \prime}\right) c$, where $z^{\prime}=z+0.0405$ and $z^{\prime \prime}$ is the coordinate value in the hypothetical paraelectric state. The xyz coordinates are given in Table 1.

|  | Wyckoff position | $z^{\prime}$ | $z^{\prime \prime}$ | $\Delta z(\AA)$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Pb}(1)$ | 16(d) | 0.0405 | 0 | $0 \cdot 300$ |
| $\mathrm{Pb}(2)$ | 4 (b) | 0.1915 | $0 \cdot 25$ | -0.433 |
| $\mathrm{Cr}(1)$ | 8 (c) | 0.0295 | 0 | 0.219 |
| $\mathrm{Cr}(2)$ | 4(a) | $0 \cdot 2205$ | 0.25 | -0.219 |
| F(1) | 16(d) | -0.1325 | -0.1695 | 0.274 |
| F(2) | 16(d) | 0.2065 | 0.1695 | 0.274 |
| F(3) | 16(d) | 0.7285 | 0.75 | -0.159 |
| F(4) | 8 (c) | 0.0975 | 0 | 0.722 |
| F(5) | 8 (c) | 0.0595 | 0 | 0.441 |
| F(6) | 8 (c) | -0.0425 | 0 | -0.315 |
| F(7) | 4(a) | 0.4725 | 0.5 | -0.204 |

atomic parameters are listed in Table 1.* Reversal of the polarization direction led to a significant increase in the value of $w R$. Atomic scattering factors for $\mathrm{Pb}^{2+}, \mathrm{Cr}^{3+}$ and $\mathrm{F}^{-}$, including anomalous dispersion, were taken from International Tables for $X$-ray Crystallography (1974).

## Atomic reversal of the spontaneous polarization

A shift of origin along the polar $c$ axis by -0.0405 has the effect of placing $\operatorname{Cr}(1)$ and $\operatorname{Cr}(2)$ symmetrically between the planes $z=0$ and $z=\frac{1}{4}$, resulting in the location of $\mathrm{Cr}(1)$ at $0.219 \AA$ above $z=0$ and $\mathrm{Cr}(2)$ at the same distance below $z=\frac{1}{4}$. A transformation of the $x y z$ coordinates in Table 1 to this origin is convenient when discussing the atomic model that underlies the reversal of spontaneous polarization sense in $\mathrm{Pb}_{5} \mathrm{Cr}_{3} \mathrm{~F}_{19}$. Any other origin increases $\operatorname{Cr}(\Delta z)_{\text {max }}$ and hence the predicted value of $T_{c}$, see below. Table 2 contains the corresponding $z^{\prime}$ coordinates for all atoms, where $z^{\prime}=z+0 \cdot 0405$. Table 2 also lists the $z^{\prime \prime}$ values of the $x y z^{\prime \prime}$ coordinates for each atom in the hypothetical paraelectric phase: this phase corresponds to space group $14 / \mathrm{mcm}$ for all atoms except $\mathrm{F}(1)$ and $\mathrm{F}(4)$, which have $z^{\prime \prime}$ coordinates of equal magnitude but opposite sign. The multiplicity and location of each atom at $x y z^{\prime \prime}$ correspond to zero polarization. A structural determination of this phase is in progress and will be reported later.

The magnitudes ( $\AA$ ) of the displacements $\Delta z$ between $z^{\prime}$ and $z^{\prime \prime}$ are also given in Table 2. All are less than $0.72 \AA$, well within the range expected for ferroelectric crystals (Abrahams, 1988). A change in

[^1]the sign of all $\Delta z$ gives an equivalent atomic arrangement but with a spontaneous polarization opposite in sense to that implied by the $x y z^{\prime}$ coordinates, as required for a demonstration of structural ferroelectricity.

The temperature at which the ferroelectric phase transforms to the paraelectric phase may be estimated from the magnitude of the Cr -atom $\Delta z$ displacements, using the relationship in (1) (Abrahams, Kurtz \& Jamieson, 1968), since Cr forms the shortest and least ionic bonds with F in this structure:

$$
\begin{equation*}
T_{c}=(\not / / 2 k)(\Delta z)^{2} \mathrm{~K}, \tag{1}
\end{equation*}
$$

where $\mathscr{K}^{k}$ is a force constant and $k$ is Boltzmann's constant. $z / 2 k \simeq 2 \times 10^{4} \mathrm{~K} \AA^{-2}$. The uncorrected value of $\Delta z$ for Cr as given in Table 2 corresponds to $T_{c}=950 \mathrm{~K}$. The atomic displacements of Cr , however, should be considered in relation to the displacements of the F atoms that form distorted octahedra around them. The Cr atoms are significantly displaced along the polar direction from the centers of the $\operatorname{Cr}(1) \mathrm{F}_{6}$ and $\operatorname{Cr}(2) \mathrm{F}_{6}$ octahedra. The displacements are respectively 0.030 (1) and 0.043 (1) $\AA$. Correction of $\Delta z$ by these values gives $\Delta z=0.189$ (9) $\AA$ for $\mathrm{Cr}(1)$ and 0.176 (4) $\AA$ for $\mathrm{Cr}(2)$, for a weighted-mean displacement of $0 \cdot 178$ (4) $\AA$ since these two values do not differ significantly. The corresponding Curie temperature predicted from (1) is hence $635(30) \mathrm{K}$, in satisfactory agreement with the experimental $T_{c}=555 \mathrm{~K}$ (Arquis-Canouet, Ravez \& Abrahams, 1986).

It may be noted from Table 2 that the polar displacement of $\mathrm{Cr}(1)$ is opposite in sense to that of $\mathrm{Cr}(2)$, hence it is likely that the spontaneous polarization $P_{s}$ will be considerably less than that estimated from the mean corrected value of $\Delta z$ using the relation

$$
\begin{equation*}
P_{s}=(258 \pm 9) \times 10^{-2} \Delta z \mathrm{C} \mathrm{~m}^{-2} \tag{2}
\end{equation*}
$$



Fig. 1. The four independent metal atom-fluorine polyhedra in $\mathrm{Pb}_{5} \mathrm{Cr}_{3} \mathrm{~F}_{19}$ and their interconnections as viewed down the polar axis. Atom $\mathrm{Cr}(2)$ is located on the fourfold axis and $\mathrm{Pb}(2)$ at the mirror-plane intersection. The thermal ellipsoids for all atoms except $\mathrm{F}(7)$ are outlined with $50 \%$ probability. The $a$ axis is horizontal, the $b$ axis vertical.

Table 3. Coordination distances $(\AA)$ in $\mathrm{Pb}_{5} \mathrm{Cr}_{3} \mathrm{~F}_{19}$
The superscripts (i)-(x) denote the following equivalent coordinates with $x, y, z$ taken from Table 1: (i) $\frac{1}{2}-x, \frac{1}{2}-y, \frac{1}{2}+z$; (ii) $1-$ $y, x, z$; (iii) $y, x, z-\frac{1}{2}$; (iv) $\frac{1}{2}+x, \frac{1}{2}-y, z-1$; (v) $\frac{1}{2}-x, \frac{1}{2}-y, z-\frac{1}{2}$; (vi) $\frac{1}{2}-y, \frac{1}{2}+x, z-\frac{1}{2}$; (vii) $\frac{1}{2}+x, \frac{1}{2}-y, z$; (viii) $-x, y, \frac{1}{2}+z$; (ix)

$$
-y, x, z ; \text { and }(\mathrm{x}) x, y, z-\frac{1}{2}
$$

| $\mathrm{Pb}(1)-\mathrm{F}(5)$ | $2.357(9)$ | $\mathrm{Pb}(2)-\mathrm{F}(6)$ | $2.267(26) \times 2$ |
| :---: | :--- | :---: | :--- |
| $-\mathrm{F}(1)^{1}$ | $2.496(22)$ | $-\mathrm{F}(6)^{\text {nin }}$ | $2.458(28) \times 2$ |
| $-\mathrm{F}(4)$ | $2.565(5)$ | $-\mathrm{F}(5)^{1 \times}$ | $2.521(12) \times 2$ |
| $-\mathrm{F}(2)^{\text {II }}$ | $2.619(12)$ | $-\mathrm{F}(2)$ | $3.025(10)^{*} \times 4$ |
| $-\mathrm{F}(2)^{\text {in }}$ | $2.700(12)$ |  |  |
| $-\mathrm{F}(3)^{\text {iv }}$ | $2.830(24)$ |  |  |
| $-\mathrm{F}(3)^{v}$ | $2.837(18)$ |  |  |
| $-\mathrm{F}(3)^{\text {ii }}$ | $2.879(19)$ |  |  |
| $-\mathrm{F}(1)^{\text {vi }}$ | $2.932(18)^{*}$ |  |  |
| $\mathrm{Cr}(1)-\mathrm{F}(4)$ | $1.881(10)$ | $\mathrm{Cr}(2)-\mathrm{F}(7)^{x}$ | $1.834(56)$ |
| $-\mathrm{F}(1)$ | $1.912(20) \times 2$ | $-\mathrm{F}(3)^{x}$ | $1.851(14) \times 4$ |
| $-\mathrm{F}(6)$ | $1.932(14)$ | $-\mathrm{F}(7)$ | $1.870(56)$ |

*Next largest $\mathrm{Pb}(1)-\mathrm{F}$ distance is $3.302(17) \AA$, next largest $\mathrm{Pb}(2)-\mathrm{F}$ distance is 3.588 (21) $\AA$.

Table 4. Average values of $\mathrm{Cr}-\mathrm{F}$ distances $(\AA)$ in $\mathrm{Cr}^{3+}$ compounds determined after 1970

| Compound | Average distance* | Reference $\dagger$ |
| :---: | :---: | :---: |
| $\mathrm{NH}_{4} \mathrm{CrF}_{4}$ | 1.886 (21) | ZNBAD 34934 |
| $\mathrm{NaBaCr}_{2} \mathrm{~F}_{9}$ | 1.892 (43), 1.897 (45) | JSSCB 56288 |
| $\mathrm{KPbCr}_{2} \mathrm{~F}_{9}$ | 1.898 (22), 1.900 (25) | JSSCB 41272 |
| $\mathrm{Cs}_{2} \mathrm{NaCrF}_{6}$ | $\begin{aligned} & 1.899[5], 1.906[6] \\ & 1.906[5], 1.913[6] \end{aligned}$ | $\begin{aligned} & \text { JSSCB } 1839 \\ & \text { ZENBA } 30462 \end{aligned}$ |
| $\mathrm{Rb}_{2} \mathrm{CrF}_{5}$ | 1.901 (67) | ACBCA 302688 |
| $\mathrm{BaCrF}_{5}$ | 1.902 (33) | ZNBAD 3754 |
| $\mathrm{CaCrF}_{5}$ | $1 \cdot 902$ (43), 1-908 (71) | MRBUA 6561 |
|  |  | MRBUA 8593 |
| $\mathrm{BaLiCrF}_{6}$ | 1.903 (22) | ZAACA 40623 |
| $\left(\mathrm{N}_{2} \mathrm{H}_{5}\right)_{3} \mathrm{CrF}_{6}$ | 1.905 (26) | ACBCA 282028 |
| $\mathrm{CsCrF}_{4}$ | 1.907 (43) | ZAACA 442151 |
| $\mathrm{NaCrF}_{4}$ | 1.908 (37) | JRPSD 1979213 |
| $\mathrm{Rb}_{2} \mathrm{NaCrF}_{6}$ | 1.936 [20] | ZAACA 391117 |
| $\mathrm{Cs}_{2} \mathrm{KCrF}_{6}$ | 1.936 [10] | ZAACA 391117 |
| $\mathrm{Rb}_{2} \mathrm{KCrF}_{6}$ | 1.940 [10] | ZAACA 391117 |

*See footnote at the bottom of this page for e.s.d.'s of average distance as given in parentheses; square brackets denote the author's own e.s.d. for uniquely determined distances.
$\dagger$ See Notes for Authors (1983) for a description of journal codens.
(Abrahams et al., 1968), which results in $P_{s} \simeq$ $0.46 \mathrm{C} \mathrm{m}^{-2}$.

## Description of the structure of $\mathbf{P b}_{\mathbf{5}} \mathbf{C r}_{\mathbf{3}} \mathbf{F}_{\mathbf{1 9}}$

The structure of $\mathrm{Pb}_{5} \mathrm{Cr}_{3} \mathrm{~F}_{19}$ consists of connected $\mathrm{PbF}_{9}, \mathrm{PbF}_{10}$ and $\mathrm{CrF}_{6}$ polyhedra that share corners, edges and faces. The four independent polyhedral types and their interconnections are illustrated in Fig. 1. The resulting grouping may be considered as the building block of the structure below 555 K . $\mathrm{Cr}(1)$ and $\mathrm{Cr}(2)$ form distorted octahedra with average $\mathrm{Cr}-\mathrm{F}$ distances of 1.922 (25) and 1.851 (9) $\AA$, respectively, see Table $3^{*}$ and next section. As

[^2]expected from a consideration of the larger e.s.d. for $\mathrm{Cr}(1)-\mathrm{F}$, that octahedron is more irregular, with a maximum angular distortion of $9.3^{\circ}$ from regularity as compared with $3 \cdot 5^{\circ}$ for the $\mathrm{Cr}(2)$ octahedron. The $\mathrm{Cr}^{3+}-\mathrm{F}$ distance given by Shannon's (1976) ionic radii for coordination numbers $6(\mathrm{Cr})$ and $3(\mathrm{~F})$ is $1 \cdot 915 \AA$. A survey of recent reports of this distance based on determinations made later than 1970 on a variety of mixed $3 d$-metal fluoride structures, as contained in the Inorganic Crystal Structure Database (Crystallographic Databases, 1987), is summarized in Table 4. The average $\mathrm{Cr}(1)-\mathrm{F}$ distance is normal with respect to Table 4, that of $\mathrm{Cr}(2)-\mathrm{F}$ is somewhat shorter.

Nine-coordinated $\mathrm{Pb}(1)$, see Table 3, forms a distorted polyhedron that may be described as a tricapped trigonal prism with $\mathrm{F}(1) \mathrm{F}(4) \mathrm{F}(5)$ and $\mathrm{F}(\mathrm{I}) \mathrm{F}(3) \mathrm{F}(3)$ forming the triangular prism faces, as outlined in Fig. 1. The average $\mathrm{Pb}(1)-\mathrm{F}$ distance is $2 \cdot 69(20) \AA . \mathrm{Pb}(2)$ is coordinated by four $\mathrm{F}(6)$ and two $\mathrm{F}(5)$ atoms, with an average $\mathrm{Pb}-\mathrm{F}$ distance of 2.42 (8) $\AA$ and with four additional $F(2)$ atoms at

Table 5. Unit-cell data for $\mathrm{Pb}_{5} \mathrm{Cr}_{3} \mathrm{~F}_{19}, \mathrm{~Pb}_{5} \mathrm{~W}_{3} \mathrm{O}_{9} \mathrm{~F}_{10}$, $\mathrm{Sr}_{3}\left(\mathrm{FeF}_{6}\right)_{2}$ and $\mathrm{SrAlF}_{5}$

|  | $\mathrm{Pb}_{5} \mathrm{Cr}_{3} \mathrm{~F}_{19}$ | $\mathrm{~Pb}_{5} \mathrm{~W}_{3} \mathrm{O}_{9} \mathrm{~F}_{10}$ | $\mathrm{Sr}_{3}\left(\mathrm{FeF}_{6}\right)_{2}$ | $\mathrm{SrAlF}_{5}{ }^{*}$ |
| :--- | :---: | :---: | :---: | :---: |
| $a(\AA)$ | $14.384(5)$ | $14.583(3)$ | $20.338(2)$ | $14.09(2)$ |
| $c(\AA)$ | $7.408(2)$ | $7.365(1)$ | $14.668(2)$ | $14.33(2)$ |
| Space group | $I 4 \mathrm{~cm}$ | 14 | $14_{1}$ | $P 4$ |
| $Z$ | 4 | 4 | 24 | 32 |
| Unit-cell | $\mathrm{Pb}_{20} \mathrm{Cr}_{12} \mathrm{~F}_{76}$ | $\mathrm{~Pb}_{20} \mathrm{~W}_{12} \mathrm{O}_{36} \mathrm{~F}_{40}$ | $\mathrm{Sr}_{18} \mathrm{Fe}_{12} \mathrm{~F}_{72}$ | $\mathrm{Sr}_{16} \mathrm{Al}_{16} \mathrm{~F}_{80}$ |
| content $\dagger$ <br> Reference | Arquis-Canouet, | Abrahams | von der Mühll | von der Mühll |
|  | Ravez \& Abrahams | et al. $(1987)$ | $(1974)$ | et al. (1971) |
|  | $(1986)$ |  |  |  |

*Structure determination was reported in subcell with $c^{\prime}=c / 2$ and space group 14.
$\dagger$ Calculated for a unit-cell volume equivalent to that of the $\mathrm{Pb}_{5} \mathrm{Cr}_{3} \mathrm{~F}_{19}$ cell.
3.024 (1) $\AA$. Taking the coordination number as ten, the resulting polyhedron may be described as bisdisphenoidal, with the four $F(6)$ atoms forming an elongated disphenoid (tetrahedron) and the two $\mathrm{F}(5)$ and two pairs of $F(2)$ atoms occupying the place of the flattened disphenoid. The distance between $\mathrm{Pb}(2)$ and the midpoint of the $\mathrm{F}(2)$ pair is $2 \cdot 72(2) \AA$. Shinn


Fig. 2. Projections on the $a b$ plane of the structures of $(a) \mathrm{Pb}_{5} \mathrm{Cr}_{3} \mathrm{~F}_{19}$, (b) $\mathrm{Pb}_{5} \mathrm{~W}_{3} \mathrm{O}_{9} \mathrm{~F}_{10}$, (c) $\mathrm{Sr}_{3}\left(\mathrm{FeF}_{6}\right)_{2}$ and (d) $\mathrm{SrAlF}_{5}$. Views of $\mathrm{Sr}_{3}\left(\mathrm{FeF}_{6}\right)_{2}$ and $\mathrm{SrAlF}_{5}$ are drawn slightly idealized in space groups 14 , and 14 , respectively, with the $\mathrm{Pb}_{5} \mathrm{Cr}_{3} \mathrm{~F}_{19}$-equivalent unit cells outlined. The octahedra about $\mathrm{Cr}, \mathrm{W}, \mathrm{Fe}$ or Al are shown shaded. The larger circles O and denote Pb or Sr at $z=0$ and $\frac{1}{2}$, respectively, while ( denotes Pb at $z=0.15$ and 0.65 in $(a), \mathrm{Pb}$ at $z=0.25$ and 0.75 in $(b)$ and Sr at 0.23 and 0.73 in $(c)$. The smaller circles $\circ$ and $\bullet$ represent $F(5)$ in $(a), F(2)$ in $(b)$ and $F(11)$ in (c), see references in Table 5.
\& Eick (1968) found a similar coordination polyhedron in $\mathrm{La}_{2}\left(\mathrm{CO}_{3}\right)_{3} \cdot 8 \mathrm{H}_{2} \mathrm{O}$. The $\mathrm{Pb}^{2+}-\mathrm{F}$ distances as given by Shannon (1976) are $2.70 \AA$ for tencoordination, $2.65 \AA$ for nine-coordination, and $2 \cdot 49 \AA$ for six-coordination. A wide range of $\mathrm{Pb}^{2+}-\mathrm{F}$ distances has been reported in the literature and the values determined for $\mathrm{Pb}_{5} \mathrm{Cr}_{3} \mathrm{~F}_{19}$ lie well within the average distribution.

## Structural relationships among the $\mathbf{S r A l F}_{5}$-like ferroelectrics

The four structural families with formulas $A B \mathrm{~F}_{5}(A$ $=\mathrm{Sr}, B=\mathrm{Al}, \mathrm{Cr}, \mathrm{Ga} ; A=\mathrm{Ba}, B=\mathrm{Ti}, \mathrm{V}, \mathrm{Fe} ; A=$ $\mathrm{Pb}, B=\mathrm{Al}, \mathrm{Ga}$; and $A=\mathrm{Eu}, B=\mathrm{Al}), A_{3}\left(B \mathrm{~F}_{6}\right)_{2}(A=$ $\mathrm{Sr}, B=\mathrm{Fe}$; and $A=\mathrm{Pb}, B=\mathrm{Ti}, \mathrm{V}, \mathrm{Cr}, \mathrm{Fe}, \mathrm{Ga})$, $A_{5} B_{3} \mathrm{O}_{9} \mathrm{~F}_{10}(A=\mathrm{Pb}, B=\mathrm{Mo}, \mathrm{W})$ and $A_{5} B_{3} \mathrm{~F}_{19}(A=$ $\mathrm{Pb}, B=\mathrm{Cr}, \mathrm{Al}$; and $A=\mathrm{Sr}, \mathrm{Ba}, B=\mathrm{Ti}, \mathrm{V}, \mathrm{Cr}, \mathrm{Fe}$, Ga ) are closely interrelated (Abrahams et al., 1987). The simple relationships among the unit-cell dimensions for the archetypes of each are noted in the Introduction; numerical values are given in Table 5.

Each family crystallizes with tetragonal symmetry but in a different space group. The equivalent formulas for each archetype, normalized to a unit cell with volume comparable to that of $\mathrm{Pb}_{5} \mathrm{Cr}_{3} \mathrm{~F}_{19}$ as listed in Table 5, show significant differences and similarities. Thus, the total number of $A+B$ atoms in the $\mathrm{Pb}_{5} \mathrm{Cr}_{3} \mathrm{~F}_{19}$ family (32) is identical to that in the $\mathrm{Pb}_{5} \mathrm{~W}_{3} \mathrm{O}_{9} \mathrm{~F}_{10}$ and $\mathrm{SrAlF}_{5}$ families but with a different $A, B$ distribution in the latter, whereas the total number of $A+B$ atoms in the $\mathrm{Sr}_{3}\left(\mathrm{FeF}_{6}\right)_{2}$ family is fewer by two. Moreover, $\mathrm{Pb}_{5} \mathrm{Cr}_{3} \mathrm{~F}_{19}$ has four fewer F atoms than $\mathrm{SrAlF}_{5}$ and four more than $\mathrm{Sr}_{3}\left(\mathrm{FeF}_{6}\right)_{2}$ in the equivalent unit cell. It may also be noted that, while the role of oxygen in the $\mathrm{Pb}_{5} \mathrm{~W}_{3} \mathrm{O}_{9} \mathrm{~F}_{10}$ family is similar to that of fluorine in $\mathrm{Pb}_{5} \mathrm{Cr}_{3} \mathrm{~F}_{19}$ with an identical total number of O and F atoms in the equivalent unit cell, the presence of O atoms results in a space-group change.

Fig. 2 shows the structures of (a) $\mathrm{Pb}_{5} \mathrm{Cr}_{3} \mathrm{~F}_{19}$, (b) $\mathrm{Pb}_{5} \mathrm{~W}_{3} \mathrm{O}_{9} \mathrm{~F}_{10}$, (c) $\mathrm{Sr}_{3}\left(\mathrm{FeF}_{6}\right)_{2}$ and (d) $\mathrm{SrAlF}_{5}$ projected along each $c$ axis. The octahedra about $\mathrm{Cr}, \mathrm{W}, \mathrm{Fe}$ and Al , respectively, are shown together with the positions of the Pb or Sr atoms. Figs. 2(a), 2(b) and 2(c) show, in addition, the positions of F atoms not coordinated to $\mathrm{Cr}, \mathrm{W}$ or Fe . The four structures are very similar although $\mathrm{Pb}_{5} \mathrm{Cr}_{3} \mathrm{~F}_{19}$, which crystallizes in space group 14 cm , is noticeably more symmetrical than the others. The major difference between structures is that $\mathrm{SrAlF}_{5}$ has two kinds of corner-sharing chains of $B \mathrm{~F}_{6}$ octahedra whereas the three other
families have only one. The $B \mathrm{~F}_{6}$ chains along $x=\frac{1}{2}, y$ $=0$ and $x=0, y=\frac{1}{2}$ in $\mathrm{SrAlF}_{5}$ are replaced by chains of eight- and ten-coordinated Pb in $\mathrm{Pb}_{5} \mathrm{~W}_{3} \mathrm{O}_{9} \mathrm{~F}_{10}$ and $\mathrm{Pb}_{5} \mathrm{Cr}_{3} \mathrm{~F}_{19}$, respectively, and by chains of eightcoordinated Sr along $x=\frac{1}{2}, y=0$ in $\mathrm{Sr}_{3}\left(\mathrm{FeF}_{6}\right)_{2}$ (transformed to the same unit cell as $\mathrm{Pb}_{5} \mathrm{Cr}_{3} \mathrm{~F}_{19}$ ). A corresponding chain along $x=0, y=\frac{1}{2}$ is missing in the latter compound.

Following completion of this work, we learned (LeBail, 1989) that $\mathrm{Pb}_{5} \mathrm{Fe}_{3} \mathrm{~F}_{19}$ is isostructural with $\mathrm{Pb}_{5} \mathrm{Cr}_{3} \mathrm{~F}_{19}$ but that, owing to the high displacement anisotropy of $\mathrm{Pb}^{2+}$, the structure determination of the former has not been published.

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## References

Abrahams, S. C. (1988). Acta Cryst. B44, 585-595.
Abrahams, S. C., Bernstein, J. L. \& Keve, E. T. (1971). J. Appl. Cryst. 4, 284-290.
Abrahams, S. C., Kurtz, S. K. \& Jamieson, P. B. (1968). Phys. Rev. 172, 551-553.
Abrahams, S. C., Marsh, P. \& Ravez, J. (1987). J. Chem. Phys. 87, 6012-6020.
Abrahams, S. C., Ravez, J., Canouet, S., Grannec, J. \& LoiaCONO, G. M. (1984). J. Appl. Phy's. 55, 3056-3060.
Abrahams, S. C., Ravez, J., Simon, A. \& Chaminade, J. P. (1981). J. Appl. Phys. 52, 4740-4743.

Arquis-Canouet, S., Ravez, J. \& Abrahams, S. C. (1986). J. Appl. Cryst. 19, 374-376.
Arquis-Canouet, S., Ravez, J., Chaminade, J. P., Hagenmuller, P., Abrahams, S. C. \& Marsh, P. (1986). J. Appl. Phys. 60, 357-360.
Busing, W. R., Martin, K. O. \& Levy, H. A. (1973). J. Appl. Cryst. 6, 309-346.
Crystallographic Databases (1987). Chester: International Union of Crystallography.
International Tables for Crystallography (1987). Vol. A. Dordrecht: Kluwer Academic Publishers.
International Tables for X-ray Crystallography (1974). Vol. IV. Birmingham: Kynoch Press. (Present distributor Kluwer Academic Publishers, Dordrecht.)
LeBail, A. (1989). Private communication.
MÜHll, R. von der (1974). C. R. Acad. Sci. 278, 713-715.
Mühll, R. von der, Andersson, S. \& Galy, J. (1971). Acta Cryst. B27, 2345-2353.
Notes for Authors (1983). Acta Cryst. A39, 174-186.
Ravez, J., Abrahams, S. C., Marsh, P., Arquis, S. \& Chaminade, J. P. (1985). Jpn. J. Appl. Phys. 24(Suppl. 2), 232.
Ravez, J., Arquis, S., Grannec, J., Simon, A. \& Abrahams, S. C. (1987). J. Appl. Phys. 62, 4299-4301.

Ravez, J., von der Mûhll, R. \& Hagenmuller, P. (1975). J. Solid State Chem. 14, 20-24.
Shannon, R. D. (1976). Acta Cryst. A32, 751-767.
Shinn, D. B. \& Eick, H. E. (1968). Inorg. Chem. 7, 1340-1345.


[^0]:    © 1990 International Union of Crystallography

[^1]:    * Lists of structure factors and anisotropic harmonic (all atoms) displacement and anharmonic $[\mathrm{Pb}, \mathrm{Cr}$ and $\mathrm{F}(7)$ atoms $]$ tensor coefficients have been deposited with the British Library Document Supply Centre as Supplementary Publication No. SUP 52906 ( 23 pp .). Copies may be obtained through The Technical Editor, International Union of Crystallography, 5 Abbey Square, Chester CH1 2HU, England.

[^2]:    * The standard deviations of these and other average $\langle x\rangle=$ $(1 / n) \sum x_{i}$ distances are estimated as $\left\{[1 /(n-1)] \sum\left(x_{i}-\langle x\rangle^{2}\right)\right\}^{1 / 2}$.

